# Scheme for remote implementation of partially unknown quantum operation of two qubits in cavity QED

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By constructing the recovery operations of the protocol of remote implementation of partially unknown quantum operation of two qubits [An Min Wang: PRA, **74**, 032317(2006)], we present a scheme to implement it in cavity QED. Long-lived Rydberg atoms are used as qubits, and the interaction between the atoms and the field of cavity is a nonresonant one. Finally, we analyze the experimental feasibility of this scheme.

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## I. INTRODUCTION

The remote implementation of quantum operation (RIO) is defined as that a quantum operation performed on the local system(Alice's one) is teleported and acts on an unknown state belonging to the remote system (Bob's) [1, 2, 3, 4]. In Ref.[1] and subsequent research[3], the authors conclude that, the optimal LOCC(local operation and classical communication) procedure to implement remotely an arbitrary unitary operator $U$ on a qudit with the shared entanglement is by the means of "bidirectional state teleportation". Furthermore, the remote implementation of a unitary transformation on the state of a qubit is studied[2]. Just as the teleportation [5] of an unknown quantum state, in the process of RIO, entangled states are used. However, the cost of entanglement resources is dependent on whether quantum operations are unknown or partially unknown (known). When it comes to the "partially unknown operation", it implies that the quantum operation satisfies some given restricted conditions. As in the reference [2], the authors considere the case of two kinds of one-qubit operations, one of them only has non-zero diagonal elements: arbitrary rotations around a fixed direction $\overrightarrow{n}$, the other just has non-zero offdiagonal elements: it a $\pi$ rotation about an arbitrary direction lying in a plane orthogonal to $\overrightarrow{n}$. For the cases more than one qubit, for example, N qubits case, Wang [4] proves that the quantum operations only with one non-zero element in every row and every column of their representation matrices can be able to be implemented remotely in a faithful and determined way, if we only have N $e$-bits and use Hadamard gates to transfer the effect of operation to Bob's qubits. In this paper, our motivation is just to present a scheme of remote implementation of partially unknown quantum operations of two qubits based on the well-known technology and method. Recently, by using a linear optics set-up, a remote rotation by 120° about the $z$ axis has been implemented experimentally on the target photons[6]. Moreover, the authors claim that the scheme can be generalized to implement the single qubit subsets discussed in[2].

Cavity QED [7, 8], optical systems [9], ion trap [10] and NMR [11] are all used to demonstrate quantum information processing and quantum computation. Recently, cavity QED technology has attracted a lot of interest. In this context, cavity QED with circular Rydberg atoms and superconducting cavities presents a peculiar interest. In cavity QED, quantum logic gates are constructed [8]; Bell-state [12, 13, 14], GHZ state(W state) [15], even the $n$-particle entangled state [16] are generated. Some important tasks of quantum information processing and quantum communication, such as teleportation [17, 18], quantum state sharing [19] and Grover's search ([20] and the references in) are successfully implemented by using cavity QED. Some of experimental demonstrations of quantum information and quantum computation in cavity QED have also been proposed [21, 22].

In this paper, by constructing the recovery operations of remote implementation of two-qubit partially unknown quantum operations, we find it is possible to carry out the quantum information processing in cavity QED. Then we present the scheme in cavity QED and analyze its experimental feasibility. The remainder of this paper is organized as follows: In Sec.II, a simple introduction of Wang's protocol is presented and the recovery operations are constructed; in Sec.III, the scheme of remote implementation of two-qubit partially unknown quantum operation in cavity QED is presented; finally, in Sec.IV, we present the discussion and conclusion.

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## II. REMOTE IMPLEMENTATION OF TWO-QUBIT PARTIALLY UNKNOWN QUANTUM OPERATIONS

The structure of the two-qubit partially unknown quantum operations which can be remotely implemented and the recovery operation are presented in Ref.[4]. Because there is only one nonzero element in every row and every column of the matrices of the two-qubit operations, there are 24 kinds of operations. They can be written as:

$$T_2(x,t) = \sum_{m=00}^{11} t_m |m\rangle\langle p_m(x)| = \begin{pmatrix} t_{00} & 0 & 0 & 0 \\ 0 & t_{01} & 0 & 0 \\ 0 & 0 & t_{10} & 0 \\ 0 & 0 & 0 & t_{11} \end{pmatrix} R_2(x), \tag{1}$$

where $p_m(x)$ is the corresponding elements of $p(x)$, which is the permutations of the list $(00, 01, 10, 11)$ and $x = 1, 2, \cdots, 24$. So $p(x) = (p_{00}(x), p_{01}(x), p_{10}(x), p_{11}(x)) = (00, 01, 10, 11), (00, 01, 11, 10), \cdots, (11, 10, 01, 00)$. A part of the recovery operation:

$$R_2(x) = T_2(x, 0) = \sum_{m=00}^{11} |m\rangle\langle p_m(x)|, \tag{2}$$

as the above $x = 1, 2, \cdots, 24$. The recovery operations correspond to the two-qubit quantum operations by Alice sends the $x$ to Bob with 5 bits.

We briefly recall the remote implementation of two-qubit partially unknown quantum operation [4]. Two entangled states $|\Psi^+\rangle_{A_1B_1}, |\Psi^+\rangle_{A_2B_2}$ work as the channel, qubits $A_1, A_2$ belong to Alice and $B_1, B_2$ belong to Bob. Another two qubits $Y_1, Y_2$ in an unknown state $|\xi\rangle_{Y_1Y_2}$ belong to Bob too. Two-qubit partially unknown quantum operations acted by Alice can work on qubits $B_1, B_2$ as follows. Bob first performs two controlled-NOT ($C_{\rm not}$) transformations by using $Y_1, Y_2$ as control qubits and $B_1, B_2$ as target qubits, respectively. Then he measures his qubits $B_1$ and $B_2$ in the computational basis $|b_1\rangle_{B_1}\langle b_1| \otimes |b_2\rangle_{B_2}\langle b_2|$, where $b_1, b_2 = 0, 1$ and sends the results to Alice. After receiving the two bits, Alice carries out the quantum operations $\sigma_{b_1}^{A_1} \otimes \sigma_{b_2}^{A_2}$ on her two qubits $A_1, A_2$ respectively. Subsequently, she acts $T_2(x,t)$ on $A_1A_2$ and executes two Hadamard gate transformation $H_{A_1} \otimes H_{A_2}$. In the end, she measures her two qubits in the basis $|a_1\rangle_{A_1}\langle a_1| \otimes |a_2\rangle_{A_2}\langle a_2| (a_1, a_2 = 0, 1)$ and sends the results and $x$ to Bob. As having been mentioned, the transmission of $x$ is to let Bob choose $R_2(x)$ correctly. With this information, Bob's recovery operations are

$$\mathcal{R}(a_1, a_2, x) = \{[(1-a_1)\sigma_0 + (a_1\sigma_3)]_{Y_1} \otimes [(1-a_2)\sigma_0 + (a_2\sigma_3)]_{Y_2}\} R_2(x)_{Y_1Y_2}. \tag{3}$$

With this steps, the two-qubit partially unknown operations can be remotely implemented on qubits $Y_1Y_2$.

The possible obstacle to demonstrate the protocol is the recovered operation $R_2(x)$. Fortunately, we can construct $R_2(x)$ by using the $C_{\rm not}$ gate and the NOT gate $\sigma_x$. Actually, this comes from the fact that any multiqubit logic gate can be decomposed as $C_{\rm not}$ transformations and single qubit logic gates [23]. We have:

$$\begin{aligned}
R_2(1) &= I \otimes I, && (4)\\
R_2(2) &= C_{\rm not}(Y_1, Y_2), && (5)\\
R_2(3) &= C_{\rm not}(Y_2, Y_1) C_{not}(Y_1, Y_2) C_{\rm not}(Y_2, Y_1), && (6)\\
R_2(4) &= C_{\rm not}(Y_2, Y_1) C_{\rm not}(Y_1, Y_2), && (7)\\
R_2(5) &= C_{\rm not}(Y_1, Y_2) C_{\rm not}(Y_2, Y_1), && (8)\\
R_2(6) &= C_{\rm not}(Y_2, Y_1), && (9)\\
R_2(7) &= C_{\rm not}(Y_1, Y_2)(I \otimes \sigma_1), && (10)\\
R_2(8) &= (I \otimes \sigma_1), && (11)\\
R_2(9) &= (\sigma_1 \otimes I) C_{\rm not}(Y_1, Y_2) C_{\rm not}(Y_2, Y_1), && (12)\\
R_2(10) &= C_{\rm not}(Y_2, Y_1)(I \otimes \sigma_1), && (13)\\
R_2(11) &= C_{\rm not}(Y_2, Y_1)(\sigma_1 \otimes I) C_{\rm not}(Y_1, Y_2) C_{\rm not}(Y_2, Y_1), && (14)\\
R_2(12) &= C_{\rm not}(Y_2, Y_1) C_{\rm not}(Y_1, Y_2)(I \otimes \sigma_1), && (15)\\
R_2(13) &= C_{\rm not}(Y_2, Y_1) C_{\rm not}(Y_1, Y_2)(\sigma_1 \otimes I), && (16)\\
R_2(14) &= C_{\rm not}(Y_2, Y_1) C_{\rm not}(Y_1, Y_2)(\sigma_1 \otimes I) C_{\rm not}(Y_2, Y_1), && (17)
\end{aligned}$$

$$
\begin{aligned}
R_2(15) &= C_{\rm not}(Y_2,Y_1)(\sigma_1\otimes I), & (18)\\
R_2(16) &= C_{\rm not}(Y_1,Y_2)(\sigma_1\otimes I)C_{\rm not}(Y_2,Y_1), & (19)\\
R_2(17) &= (\sigma_1\otimes I), & (20)\\
R_2(18) &= C_{\rm not}(Y_1,Y_2)(\sigma_1\otimes I), & (21)\\
R_2(19) &= (I\otimes\sigma_1)C_{\rm not}(Y_2,Y_1), & (22)\\
R_2(20) &= C_{\rm not}(Y_1,Y_2)(I\otimes\sigma_1)C_{\rm not}(Y_2,Y_1), & (23)\\
R_2(21) &= C_{\rm not}(Y_2,Y_1)(\sigma_1\otimes I)C_{\rm not}(Y_1,Y_2), & (24)\\
R_2(22) &= C_{\rm not}(Y_2,Y_1)C_{\rm not}(Y_1,Y_2)(I\otimes\sigma_1)C_{\rm not}(Y_2,Y_1), & (25)\\
R_2(23) &= (\sigma_1\otimes I)C_{\rm not}(Y_1,Y_2), & (26)\\
R_2(24) &= (\sigma_1\otimes\sigma_1). & (27)
\end{aligned}
$$

where $C_{\rm not}(Y_1,Y_2)$ means that we use qubit $Y_1$ as the control qubit, $Y_2$ as the target qubit to do the controlled-NOT transformation, and $C_{\rm not}(Y_2,Y_1)$ means we use qubit $Y_2$ as the control qubit and qubit $Y_1$ as the target qubit. In addition, $\sigma_i$ is the Pauli matrices, with $\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ and $I$ is the $2\times2$ identity matrix.

## III. DEMONSTRATE THE PROTOCOL IN CAVITY QED

In the original protocol[4], after constructing the recovery operations, all the operations to remotely implement two-qubit partially unknown quantum operations are two-qubit $C_{\rm not}$ gate, single qubit logic gates: Hadamard gate and pauli operations. The quantum resources we need is just two $e$-bits. All of them can be realized by using cavity QED.

As we have shown in the introduction, the preparation of two-qubit entangled states has been demonstrated in many papers[12, 13, 14]. In the reference [8], Zheng and Guo proposed a realizable scheme of two-atom controlled-NOT gate in cavity QED. In the protocol, ladder-type three-level(denoted by $|g\rangle,|e\rangle$ and $|i\rangle$) atoms are used. In order to make sure that $|i\rangle$ is not affected by the atom-cavity interaction, the transition frequency between the state $|e\rangle$ and $|i\rangle$ is highly detunned from the cavity frequency.

Let us start with considering two identical ladder-type three-level atoms simultaneously interacting with a single cavity. There is no energy exchange between the atomic system and the cavity under the approximation $\delta\gg g$. In the case of the cavity field in the vacuum state, the effective Hamiltonian is given by:

$$H=\lambda\left[\sum_{j=1,2}|e_j\rangle\langle e_j|+(S_1^+S_2^-+S_1^-S_2^+)\right], \tag{28}$$

where $\lambda=g^2/\delta, S_j^+=|e_j\rangle\langle g_j|$ and $S_j^-=|g_j\rangle\langle e_j|$, with $|g_j\rangle,|e_j\rangle(j=1,2)$ being the ground and excited states of the atom. $a^+,a$ are the creation and annihilation operators of the cavity mode. $g$ is the atom-cavity coupling strength, and $\delta$ is the detuning between the atomic transition frequency $\omega_0$ and cavity frequency $\omega$.

Now, the $C_{\rm not}$ gate can be realized as follows[8]: atom 2 passes through classical field tuned to the transitions $|g\rangle\to|e\rangle$ and $|e\rangle\to|i\rangle$, the amplitudes and phases of which are appropriately chosen, so we have:

$$
\begin{aligned}
|e_2\rangle &\to \tfrac{1}{\sqrt{2}}(|e_2\rangle+|g_2\rangle)\to\tfrac{1}{\sqrt{2}}(|i_2\rangle+|g_2\rangle),\\
|g_2\rangle &\to \tfrac{1}{\sqrt{2}}(|g_2\rangle-|e_2\rangle)\to\tfrac{1}{\sqrt{2}}(|g_2\rangle-|i_2\rangle),
\end{aligned} \tag{29}
$$

after this, we send the atom 1 and 2 into the nonresonant cavity Eq.(28) simultaneously, by choosing the interaction time $\lambda t=\pi$:

$$
\begin{aligned}
|g_1g_2\rangle &\to |g_1g_2\rangle,\\
|g_1i_2\rangle &\to |g_1i_2\rangle,\\
|e_1g_2\rangle &\to |e_1g_2\rangle,\\
|e_1i_2\rangle &\to -|e_1i_2\rangle,
\end{aligned} \tag{30}
$$

then atom 2 passes through two classical fields tuned to the transitions $|e\rangle \to |i\rangle$ and $|g\rangle \to |e\rangle$ respectively by appropriately choosing the amplitudes and phases of the classical fields:

$$\begin{aligned} |g_2\rangle &\to \tfrac{1}{\sqrt{2}}(|g_2\rangle + |e_2\rangle), \\ |i_2\rangle &\to |e_2\rangle \to \tfrac{1}{\sqrt{2}}(|e_2\rangle - |g_2\rangle), \end{aligned} \tag{31}$$

with this steps, we will obtain the $C_{\rm not}$ transformation:

$$\begin{aligned} |g_1g_2\rangle &\to |g_1g_2\rangle, \\ |g_1e_2\rangle &\to |g_1e_2\rangle, \\ |e_1g_2\rangle &\to |e_1e_2\rangle, \\ |e_1e_2\rangle &\to |e_1g_2\rangle, \end{aligned} \tag{32}$$

In any physical system, single qubit gates are easily performed. To the atoms, these single qubit gates can be realized by using rotations [20]. We could also realize Hadamard gate in cavity QED by considering an atom through an initially empty resonant cavity. In the interaction picture, the Hamiltonian is [24, 25, 26]:

$$H_I = g\left[a^+S^- + aS^+\right]. \tag{33}$$

It's the Jaynes-Cummings model, where $a^+, a$ are the creation and annihilation operator of the cavity field. $S^+ = |e\rangle\langle g|, S^- = |g\rangle\langle e|$, with $|g\rangle, |e\rangle$ being the ground and excited states of the atom.

The Hadamard gate can be realized as follows: at first we send the atom through the initially empty cavity and choose the interaction time $gt = \pi$, after that we let atom cross the classical field $R_+$[27]. The process is:

$$|g\rangle \ \xrightarrow{JC}\ |g\rangle \xrightarrow{R_+} \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), \tag{34}$$

$$|e\rangle \ \xrightarrow{JC}\ -|e\rangle \xrightarrow{R_+} \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle), \tag{35}$$

where $R_+$ represents the action of the Ramsey zone

$$R_+ = \frac{1}{\sqrt{2}}(I + i\sigma_y). \tag{36}$$

Thus, in cavity QED, not only the $C_{\rm not}$ operation, but also the Hadamard gate has been realized.

Now we present the scheme to implement the remote implementation of two-qubit partially unknown quantum operations in cavity QED with three steps, where the logical states $|1\rangle$ and $|0\rangle$ are represented by atom state $|e\rangle$ and $|g\rangle$, if we use Rydberg atoms as the qubits, and the shared two Bell pairs are

$$\frac{1}{2}\left(|gg\rangle_{A_1B_1} + |ee\rangle_{A_1B_1}\right) \otimes \left(|gg\rangle_{A_2B_2} + |ee\rangle_{A_2B_2}\right), \tag{37}$$

and the unknown state of two qubits is:

$$|\xi\rangle_{Y_1Y_2} = y_{gg}|gg\rangle + y_{ge}|ge\rangle + y_{eg}|eg\rangle + y_{ee}|ee\rangle. \tag{38}$$

*Step 1*: let atom $B_1$ cross two classical fields tuned to the transitions $|g\rangle \to |e\rangle$ and $|e\rangle \to |i\rangle$ given by Eq.(29), respectively. Then, atoms $Y_1, B_1$ are sent into the nonresonant cavity simultaneously given by Eq.(28). After they pass through the cavity, atom $B_1$ crosses two classical fields tuned to the transitions $|e\rangle \to |i\rangle$ and $|g\rangle \to |e\rangle$ shown in Eq.(31)($C_{\rm not}(Y_1, B_1)$). At the same time, to the atom $B_2$ and $Y_2$, we operate them by correspondingly replacing $B_1, Y_1$ with $B_2, Y_2(C_{\rm not}(Y_2, B_2))$. Then measuring two qubits $B_1, B_2$ in the basis $|g\rangle_{B_{1(2)}}, |e\rangle_{B_{1(2)}}$ with the result $b_1, b_2$, and assuming $b_1 = b_2 = 0$, which means we get $|g\rangle_{B_1}, |g\rangle_{B_2}$, the state of the system becomes

$$\left(y_{gg}|gggg\rangle + y_{ge}|gege\rangle + y_{eg}|egeg\rangle + y_{ee}|eeee\rangle\right)_{A_1A_2Y_1Y_2} \otimes |gg\rangle_{B_1B_2}. \tag{39}$$

*Step 2*: With $b_1, b_2$, we do the rotations[17, 20] on $A_1, A_2$ so as to realize $\sigma_{b_1}^{A_1}$ and $\sigma_{b_2}^{A_2}$. Following, we act $T_2(x,t)$ on $A_1A_2$. After doing the rotations on atoms $A_1, A_2$ to realize the $H_{A_1}, H_{A_2}$, two atoms $A_1, A_2$ are measured in the basis $|g\rangle_{A_{1(2)}}, |e\rangle_{A_{1(2)}}$. External classical microwave resources resonant on the $|e\rangle - |g\rangle$ produce these rotations on the two atoms respectively. The amplitude and phase of these sources are carefully tunned to produce the required transitions. For example, if we want to remotely implement two-qubit partially unknown quantum operation

$$T_2(10,t)=\sum_{m=gg}^{ee} t_m|m\rangle\langle p_m(10)| = \left(\begin{array}{cccc} 0 & t_{gg} & 0 & 0 \\ 0 & 0 & t_{ge} & 0 \\ 0 & 0 & 0 & t_{eg} \\ t_{ee} & 0 & 0 & 0 \end{array}\right), \tag{40}$$

$p_m(10)$ is the corresponding element of $p(10)=(ge,eg,ee,gg)$. We assume that measurement outputs of atoms $A_1, A_2$ are $a_1=a_2=1$, which tell us that the atoms are in the state $|e\rangle_{A_1}, |e\rangle_{A_2}$. After Step 2, we will obtain

$$(y_{gg}t_{ee}|gg\rangle + y_{ge}t_{gg}|ge\rangle - y_{eg}t_{ge}|eg\rangle - y_{ee}t_{eg}|ee\rangle)_{Y_1Y_2} \otimes |ee\rangle_{A_1A_2} \otimes |gg\rangle_{B_1B_2}. \tag{41}$$

Now let us focus on the recovery operation.

*Step 3*: With $a_1=a_2=1$ and $x=01010$(which is used to denote decimal system 10), for the atoms $Y_1, Y_2$, after doing the rotation to realize $\sigma_1^{Y_2}$, we perform them just same as the step 1 by correspondingly substituting $Y_1, B_1$ with $Y_2, Y_1(C_{\text{not}}(Y_2,Y_1))$. Then we do rotations to implement $\sigma_3^{Y_1}, \sigma_3^{Y_2}$. Thus the system evolves into

$$T_2(10,t)\,(y_{gg}|gg\rangle + y_{ge}|ge\rangle + y_{eg}|eg\rangle + y_{ee}|ee\rangle)_{Y_1Y_2} \otimes |ee\rangle_{A_1A_2} \otimes |gg\rangle_{B_1B_2}. \tag{42}$$

With the three steps, we can remotely implement the $T_2(10,t)$ on the atoms $Y_1, Y_2$. The simple figure of the experimental apparatus is shown in Fig.1.

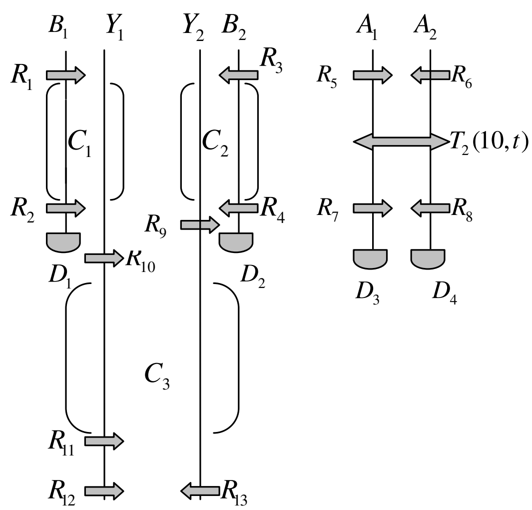


FIG. 1: Experimental apparatus. $R_i(i=1,2,\cdots,13)$ is the appropriately chosen classical field to realize the transitions among atomic levels, *Hadamard* gates and pauli operations. $C_i(i=1,2,3)$ is the nonresonant cavity, and the two atoms must be sent into it simultaneously. $D_i(i=1,2,3,4)$ is the measurement we do on atom in the basis $|g\rangle, |e\rangle$.

## IV. DISCUSSION AND CONCLUSION

We consider the experimental realization of our protocol. On the one hand, we consider the radiation of the atom. To the $C_{\text{not}}$ operation, if choosing $\delta=10g$ and $g=2\pi\times 24$kHz [8, 28], the interaction time of cavity-field is $\pi\delta/g^2 \approx 2\times 10^{-4}$s. The time needed for the atom tuned with classical field is on the order of $6.3\times 10^{-6}s$ [18], thus it is negligible at this scale[20]. So the time needed to implement the scheme is on the order of $10^{-3}$s, much shorter than the radiative time of the Rydberg atom with principal quantum numbers $49, 50$ and $51$, which is about $3\times 10^{-2}$s. On the other hand, we consider the cavity decay. With present cavity technology, a cavity with a quality factor $Q=10^8$ is experimentally achievable[28]. As discussed in [8], the photon lifetime in the cavity whose cavity frequency is about 50 GHz is $\tau_C=\frac{Q}{2\pi\nu}\approx 3\times 10^{-4}$s. In the present protocol, that the cavity is always in the vacuum state result in the suppressed cavity decay. Therefore, the cavity has only about 0.01 probability of being excited during the passage of the atoms through the cavity and the efficient decay time of the cavity is about $3 \times 10^{-2}$s, on the order of the atomic radiative time, which is much longer than the time needed to implement the scheme. In 2001, an experiment of preparing EPR pairs with two atoms using the present model has been implemented[21]. To the resonant cavity, which we use it to realize Hadamard transformation, the atom acting as the qubit must have a sufficiently long excited life-time. Luckily, the Rydberg atom with principal quantum numbers 50 and 51 is a good candidate, because the interaction time( with the same characters, we have the interaction time $\pi/g = 2 \times 10^{-5}s$) is much shorter than the atomic radiative time. Hence the time to complete the remote implementation of two-qubit partially unknown quantum operations is much shorter than that of atom radiation. However, there is still a difficulty to carry out our protocol: distinguishing the two atoms after they flying out of the cavity. Fortunately, we can use the method proposed in[22], which has developed a technique to address any specified target ion using tightly focused laser beams and by changing their internal states to "hide" the remaining ions from the target ion's fluorescence so that they are insensitive to the fluorescent light. Therefore, our scheme is realizable based on current cavity technology.

In order to realize the $C_{\rm not}$ operation, we require the two atoms are sent through a cavity simultaneously. Hence we would like to estimate the influence when the simultaneous is not exactly satisfied, that is we estimate the fidelity between the result that one atom enters the cavity sooner than the other by $0.01t$ and the Eq.(40) through an numerical calculation, and get the fidelity as high as 99.8%. The calculation results are showed in Fig.2. Therefore, our protocol are slightly affected.

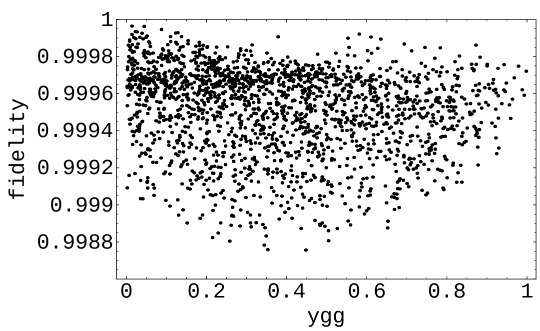


FIG. 2: Fidelity vs the value of $ygg$. In order to get the numerical results, we have assumed $t_{gg} = t_{ge} = t_{eg} = t_{ee} = x$, where $|x|^2 = 1$ resulting from the fact that $T_2(10, x)$ is a unitary transformation. In the calculation, $y_{gg}, y_{ge}, y_{eg}, aswellasy_{ee}$ are all arbitrary positive real numbers and the previous three run their values. We make sure $y_{gg}^2+y_{ge}^2+y_{eg}^2 \leq 1$, $y_{gg}^2+y_{ge}^2+y_{eg}^2+y_{ee}^2 = 1$.

In summary, we propose the scheme to remotely implement two-qubit partially unknown quantum operation in cavity QED. In order to do this, the recovery operations are constructed by $C_{\rm not}$ and single-qubit pauli operations, thus all the operations are just $C_{\rm not}$, Hadamard gate and pauli operations. We realize them and draw the conclusion that the scheme can be demonstrated in cavity QED. Besides, from the Eq.(1), the two-qubit partially unknown quantum operations can be constructed in cavity QED because we have realized $R_2(x)$.

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